Scaled momentum distributions of charged particles in dijet photoproduction at HERA

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# Scaled momentum distributions of charged particles in dijet photoproduction at HERA 

## ZEUS collaboration

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#### Abstract

The scaled momentum distributions of charged particles in jets have been measured for dijet photoproduction with the ZEUS detector at HERA using an integrated luminosity of $359 \mathrm{pb}^{-1}$. The distributions are compared to predictions based on perturbative QCD carried out in the framework of the modified leading-logarithmic approximation (MLLA) and assuming local parton-hadron duality (LPHD). The universal MLLA scale, $\Lambda_{\text {eff }}$, and the LPHD parameter, $\kappa^{\text {ch }}$, are extracted.


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## 1 Introduction

The formation of jets of hadrons can be described as a convolution of parton showering and hadronisation. Within perturbative QCD ( pQCD ), the parton shower can be described as long as the energy scale involved is sufficiently above the intrinsic scale of $\mathrm{QCD}, \Lambda_{\mathrm{QCD}}$. Hadronisation describes the process by which coloured partons become confined in colourneutral hadrons. It cannot be described within pQCD.

Perturbative QCD calculations can be performed using matrix elements up to a certain order in the strong coupling constant, $\alpha_{s}$. Alternatively, a resummation approach can be adopted, such as the modified leading-logarithmic approximation (MLLA) [1-6], where in addition to the fixed-order matrix elements, a subset of dominant terms of all orders in $\alpha_{s}$ are included. In particular, pQCD based on the MLLA can be used to predict the multiplicity and momentum spectra of partons produced within cones centred on the initial parton direction. The MLLA may only be used to describe partons at scales above some minimum cutoff, $\Lambda_{\text {eff }}>\Lambda_{\mathrm{QCD}}$. The value of $\Lambda_{\text {eff }}$ is predicted to be independent of the process considered. The local parton hadron duality (LPHD) [7] hypothesis predicts that
charged-hadron distributions should be related to the predicted parton distributions by a constant normalisation scaling factor, $\kappa^{\text {ch }}$.

Tests of the MLLA have been performed before using data from $e^{+} e^{-}$collisions at LEP [8, 9] and PETRA [10], deep inelastic scattering (DIS) ep collisions at HERA [11, 12], (anti-) neutrino-nucleon interactions from the NOMAD experiment [13] and $p \bar{p}$ collisions at the Tevatron [14]. In this analysis, the multiplicity and momentum spectra of charged hadrons within jets are studied using photoproduction $(\gamma p)$ in $e p$ collisions, in which a quasi-real photon emitted from the incoming electron collides with a proton. The events were required to have two and only two reconstructed jets and the sample was enriched in events in which the photon interacted electromagnetically as a point-like particle. The analysis probes energy scales in the range 19 to 38 GeV , which spans the energy region between those accessed previously by the ZEUS, using ep DIS collisions [11, 12], and CDF collaborations [14]. The quantities $\Lambda_{\text {eff }}$ and $\kappa^{\mathrm{ch}}$ are extracted and their universality tested.

## 2 The MLLA framework

The MLLA can describe the momentum and multiplicity spectra of partons at a specified scale, $Q_{0}$, showering from either an initial quark or gluon. The MLLA includes all terms of order $\alpha_{s}^{n} \log ^{2 n}\left(E_{\mathrm{init}}^{\mathrm{pl}}\right)$ and $\alpha_{s}^{n} \log ^{2 n-1}\left(E_{\mathrm{init}}^{\mathrm{pl}}\right)$, where $n$ is the set of positive integers and $E_{\text {init }}^{\mathrm{pl}}$ is the energy of the initial outgoing parton in the centre-of-mass frame of the incoming struck parton and exchanged photon. The "pl" superscript denotes a parton-level quantity. The MLLA accounts for colour-coherence effects between diagrams of the same order of $\alpha_{s}$ by enforcing an angular-ordering scheme [15].

The MLLA equations are only strictly valid for partons satisfying $x_{p}^{\mathrm{pl}}=\left|p_{p}^{\mathrm{pl}}\right| / E_{\text {init }}^{\mathrm{pl}} \ll 1$, where $p_{p}^{\mathrm{pl}}$ is the 3 -momentum of a parton in the centre-of-mass frame, and for roughly collinear partons emitted into a cone with a opening angle, $\theta_{c}^{\mathrm{pl}} \ll \pi$, measured with respect to the axis of the initial parton. They spectra are assumed to be dominated by gluon emissions. All partons are considered to be massless.

For the MLLA predictions used here, the singularities were regularised by a single $p_{T}^{\mathrm{rel}, \mathrm{pl}}$ cut-off at scale $Q_{0}>\Lambda_{\text {eff }}$, where $p_{T}^{\text {rel,pl }}$ is the transverse momentum with which the parton was emitted with respect to its parent. This is not the only possible way to regularise the MLLA; other forms lead to different predictions, particularly at low $x_{p}^{\mathrm{pl}}$ [16]. The MLLA neglects the different QCD radiation pattern from light- and heavy-flavour quarks, the latter of which exhibit so-called "dead-cones" [17], as the heavier quark mass screens the collinear singularity.

Predictions at the lowest valid scale, $Q_{0}=\Lambda_{\text {eff }}$, give the so-called limiting momentum spectrum of partons [18], $\bar{D}^{\text {lim,pl }}=\frac{d N^{\mathrm{pl}}}{d \xi^{\mathrm{pl}}}$, where $\xi^{\mathrm{pl}}=\ln \left(1 / x_{p}^{\mathrm{pl}}\right)$ and $N^{\mathrm{pl}}$ is the multiplicity of partons produced within a cone of opening angle, $\theta_{c}^{\mathrm{pl}}$. The predictions used here assume there to be three quark flavours, $n_{f}=3$, excited during the shower. When a value of $n_{f}$ larger than three is used instead, the theory is observed to give a poorer description of this and other data sets [18].

The shape of the predicted spectrum depends on the quantity, $Y=$ $\ln \left(E_{\text {init }}^{\mathrm{pl}} \sin \left(\theta_{c}^{\mathrm{pl}}\right) / \Lambda_{\text {eff }}\right)$. The spectrum is roughly Gaussian, although, due to the regularisation scheme adopted, falls rapidly to zero as $\xi^{\mathrm{pl}} \rightarrow Y$.

The limiting momentum spectra for quark-, $\bar{D}_{\mathrm{q}-\mathrm{jet}}^{\text {lim, }}$, and gluon-initiated, $\bar{D}_{\mathrm{g}-\mathrm{jet}}^{\mathrm{lim}, \mathrm{pl}}$, jets are related according to

$$
\begin{equation*}
\bar{D}_{\mathrm{q}-\mathrm{jet}}^{\mathrm{lim}, \mathrm{pl}}=\frac{1}{r} \bar{D}_{\mathrm{g}-\mathrm{jet}}^{\mathrm{lim}, \mathrm{pl}}, \tag{2.1}
\end{equation*}
$$

where $r=N_{\mathrm{g}-\text { jet }}^{\mathrm{pl}} / N_{\mathrm{q}-\text { jet }}^{\mathrm{pl}}$ is the ratio of parton multiplicities in gluon- and quark-initiated jets. In the MLLA, $r=C_{A} / C_{F}=9 / 4$ where $C_{A}$ and $C_{F}$ are the gluon and quark colour factors, respectively. Generally, photoproduction samples contain both gluon- and quarkinitiated jets, in the fractions denoted by $\epsilon_{\mathrm{g}}$ and $\epsilon_{\mathrm{q}}=1-\epsilon_{\mathrm{g}}$, respectively. Thus, the limiting spectrum for partons in a photoproduction sample can be parameterised as

$$
\begin{equation*}
\bar{D}^{\mathrm{lim}, \mathrm{pl}}=\left(\epsilon_{\mathrm{g}}+\frac{1-\epsilon_{\mathrm{g}}}{r}\right) \bar{D}_{\mathrm{g}-\mathrm{jet}}^{\mathrm{lim}, \mathrm{pl}} . \tag{2.2}
\end{equation*}
$$

At leading order (LO), the peak position of the limiting momentum spectrum, $\xi_{\text {peak }}^{\mathrm{pl}}$, is predicted to be at

$$
\begin{equation*}
\xi_{\mathrm{peak}}^{\mathrm{pl}}=\frac{1}{2} Y+\sqrt{c Y}-c \tag{2.3}
\end{equation*}
$$

where $c=0.29$.
Solutions to the MLLA evolution equations have also been made at so-called next-to-MLLA order. Each of these solutions partially accounts for orders not included in the equations above. With the next-to-MLLA corrections, $r$ differs from the MLLA value and has a weak dependence on $E_{\text {init }}^{\mathrm{pl}}$. In addition, the magnitudes of both $\bar{D}_{\mathrm{q}-\mathrm{jet}}^{\text {lim, } \mathrm{pl}}$ and $\bar{D}_{\mathrm{g}-\mathrm{jet}}^{\text {lim, }}$ increase by a factor, $F_{\text {nMLLA }}$, which is also weakly dependent on $E_{\text {init }}^{\mathrm{pl}}$. Next-to-MLLA $F_{\text {nMLLA }}$ and $r$ values have been used in this analysis in the same way as they were by the CDF collaboration [14], wherein more details can be found. Their values were taken from three different next-to-MLLA calculations [19-21], which differ in the way the additional orders are accounted for, leading to some spread in the predicted $F_{\text {nMLLA }}$ and $r$ values. Here, constant values of $F_{\mathrm{nMLLA}}=1.3 \pm 0.2$ and $r=1.6 \pm 0.2$ were used, with the theoretical uncertainties covering the spreads.

The LPHD approximation relates the limiting momentum spectrum of partons to that of charged hadrons within jets, $\bar{D}^{\text {lim,ch }}$, via

$$
\begin{equation*}
\bar{D}^{\mathrm{lim}, \mathrm{ch}}=\kappa^{\mathrm{ch}} \bar{D}^{\mathrm{lim}, \mathrm{pl}}=\kappa^{\mathrm{ch}}\left(\epsilon_{\mathrm{g}}+\frac{1-\epsilon_{\mathrm{g}}}{r}\right) \bar{D}_{\mathrm{g}-\mathrm{jet}}^{\mathrm{lim}, \mathrm{pl}}=K \bar{D}_{\mathrm{g}-\mathrm{jet}}^{\mathrm{lim}, \mathrm{pl}}, \tag{2.4}
\end{equation*}
$$

i.e. $K=\kappa^{\mathrm{ch}}\left(\epsilon_{\mathrm{g}}+\left(1-\epsilon_{\mathrm{g}}\right) / r\right)$. Local parton hadron duality does not account for the different fragmentation functions for light- and heavy-flavour quarks and the resulting flavour dependent differences in $\bar{D}^{\text {lim,ch }}[22,23]$. In the sample studied here, the contribution from light-flavours and charm strongly dominate that from beauty. Ignoring flavour effects and from isospin invariance, $\kappa^{\text {ch }}$ is expected to be approximately $2 / 3$. However, due to the presence of heavy-flavour quarks and the predicted dead-cones, not accounted for in the

MLLA, $\kappa^{\text {ch }}$ may be somewhat smaller [24-26]. No correction procedure was used to account for flavour dependent effects when comparing the MLLA+LPHD to the hadron-level data presented below. ${ }^{1}$

## 3 The analysis strategy

To compare the parton-level MLLA predictions to measured hadron-level data, while assuming LPHD, each variable within the MLLA had to be estimated using a related hadronlevel quantity. The hadron-level estimator for $E_{\text {init }}^{\mathrm{pl}}$ was chosen to be $E_{\text {jet }}=M_{2 j} / 2$, where $E_{\text {jet }}$ is the energy of either hadron-level jet in the dijet centre-of-mass frame and $M_{2 j}$ is the invariant dijet mass. The quantity $p_{p}^{\mathrm{pl}}$ was estimated using the momenta of the charged hadrons, $p_{\text {trk }}$. The loss of the neutral hadrons is accounted for via the LPHD factor $\kappa^{\text {ch }}$. The MLLA variable $\theta_{c}^{\mathrm{pl}}$ was estimated using the opening angle of a cone measured with respect to the reconstructed jet axis, $\theta_{c}$. Accordingly, the quantity $\bar{D}^{\text {lim,ch }}$, given in eq. (2.4), was estimated using the hadron-level multiplicity distribution of charged hadrons per jet, $N_{\text {jet }}^{\text {ch }}$, measured in bins of $E_{\text {jet }}$ and in cones of varying $\theta_{c}$, differentially in $\xi=\ln \left(E_{\text {jet }} /\left|p_{\text {trk }}\right|\right)$. These $\mathrm{d} N_{\text {jet }}^{\mathrm{ch}} / \mathrm{d} \xi$ distributions will be referred to as the $\xi$ distributions.

## 4 Experimental setup

The data analysed here were collected using the ZEUS detector during the 2005 to 2007 running periods, in which electrons ${ }^{2}$ were collided with protons with energies of $E_{e}=$ 27.5 GeV and $E_{p}=920 \mathrm{GeV}$, respectively, corresponding to a centre-of-mass energy, $\sqrt{s}=$ 318 GeV . The total sample corresponds to an integrated luminosity of $359 \pm 9 \mathrm{pb}^{-1}$. A detailed description of the ZEUS detector can be found elsewhere [28, 29]. A brief outline of the components most relevant to this analysis is given below.

Charged particles were tracked in the central tracking detector (CTD) [30-32], the microvertex detector (MVD) [33] and the straw-tube tracker (STT) [34]. The CTD and MVD were operated in a magnetic field of 1.43 T provided by a thin superconducting solenoid. The CTD drift chamber covered the polar-angle ${ }^{3}$ region $15^{\circ}<\theta<164^{\circ}$. The MVD silicon tracker consisted of a barrel (BMVD) and a forward (FMVD) section. The BMVD provided polar-angle coverage for tracks with three measurements from $30^{\circ}$ to $150^{\circ}$. The FMVD extended the polar-angle coverage in the forward region to $7^{\circ}$. The STT covered the polar-angle region $5^{\circ}<\theta<25^{\circ}$.

The high-resolution uranium-scintillator calorimeter (CAL) [35-38] consisted of three parts: the forward, the barrel and the rear calorimeters. Each part was subdivided transversely into towers and longitudinally into one electromagnetic and either one (in the rear) or two (in the barrel and forward) hadronic sections. The smallest subdivision of

[^0]the calorimeter was called a cell. The CAL relative energy resolutions, as measured under test-beam conditions, were $0.18 / \sqrt{E}$ for electrons and $0.35 / \sqrt{E}$ for hadrons, with $E$ in GeV .

## 5 Event reconstruction

A three-level trigger system was used to select events online [29, 39, 40]. At the first two levels, general characteristics of photoproduction collisions were required and background from beam-gas events was rejected. At the third level, jets were reconstructed by applying the $k_{T}$ cluster algorithm [41] to the CAL cells and a loose dijet selection was applied.

In the offline analysis, the hadronic final state was reconstructed using energy-flow objects [42, 43] (EFOs), which were formed from a combination of track and calorimeter information. This approach optimised the energy resolution and improved the one-toone correspondence between the detector-level objects and the hadrons. The EFOs were corrected to account for energy losses in the dead material and were forced to be massless by setting the energy component equal to the magnitude of the three-momentum.

Jets were reconstructed from EFOs using the $k_{T}$ cluster algorithm [41] in the longitudinally invariant inclusive mode [44] using the $p_{T}$ recombination scheme and with the $R$ parameter set to $R=1$.

Photoproduction events are characterised by the low virtuality, $Q^{2}$, of the exchanged photon. At LO, photoproduction can be categorised as being either direct, if the photon interacts as a point-like particle, or resolved, if it fluctuates into a partonic system, which then interacts with the proton. The LO direct photoproduction processes are boson gluon fusion, $\gamma g \rightarrow q \bar{q}$, and QCD Compton scattering, $\gamma q \rightarrow q g$. Important kinematic variables are the inelasticity, $y$, and the fraction of the photon momentum transferred to the hadronic final state, $x_{\gamma}$. The variable $x_{\gamma}$ can be approximated using the observable $x_{\gamma}^{\text {obs }}$, defined for a dijet event as

$$
\begin{equation*}
x_{\gamma}^{\mathrm{obs}}=\frac{\sum_{i=1}^{2} E_{T}^{\mathrm{jet}(i)} \exp \left(-\eta^{\mathrm{jet}(i)}\right)}{2 y E_{e}} \tag{5.1}
\end{equation*}
$$

where $E_{T}^{\text {jet }}$ and $\eta^{\text {jet }}$ denote the jet transverse energy and pseudorapidity in the laboratory frame, respectively. A value of $x_{\gamma}^{\text {obs }}$ approaching one indicates an event from a direct-like photoproduction process.

## 6 Event selection

To remove non-photoproduction events it was required that:

- the longitudinal position of the reconstructed vertex was in the range $\left|Z_{\mathrm{vtx}}\right| \leq 40 \mathrm{~cm}$;
- $0.2 \leq y_{\text {JB }} \leq 0.85$, where $y_{\text {JВ }}$ is the Jacquet-Blondel estimator [45] of $y$;
- no scattered electron was observed in the CAL with $E_{e}^{\prime}>5 \mathrm{GeV}$ and $y_{e}<0.85$, where $E_{e}^{\prime}$ is the energy of the scattered electron and $y_{e}$ is the electron-method estimator of $y$ [46];
- $P_{T}^{\text {miss }} / \sqrt{E_{T}} \leq 2 \mathrm{GeV}^{1 / 2}$, where $P_{T}^{\text {miss }}$ and $E_{T}$ are the reconstructed missing and total transverse momenta, respectively;
- $\left|t_{\mathrm{CAL}}^{\mathrm{top}}-t_{\mathrm{CAL}}^{\mathrm{bot}}\right|<6 \mathrm{~ns}$, where $\left|t_{\mathrm{CAL}}^{\mathrm{top}}-t_{\mathrm{CAL}}^{\text {bot }}\right|$ is the difference between the arrival times of the first signals in the top and bottom halves of the CAL;
- $N_{\text {trk }}^{\text {pri }} / N_{\text {trk }}>0.1$, where $N_{\text {trk }}^{\text {pri }} / N_{\text {trk }}$ is the ratio of the number of tracks fitted to the primary vertex to the total number of all tracks.

To select an exclusive dijet sample enriched in direct events it was required that:

- two jets were found such that:
- the highest $E_{T}^{\text {jet }}$ jet, labelled 1, had $\left|\eta^{\text {jet }}\right| \leq 1$ and $E_{T}^{\text {jet }} \geq 17 \mathrm{GeV}$;
- the second jet, labelled 2 , had $\left|\eta^{\text {jet2 }}\right| \leq 1$ and $E_{T}^{\text {jet } 2} / E_{T}^{\text {jet }} \geq 0.8$;
- the first and second jets satisfied $\left|\phi^{\text {jet1 }}-\phi^{\text {jet2 }}\right| \geq 0.9 \pi$, where $\phi^{\text {jet }}$ denotes the azimuthal angle of the jet;
- no third jet was found with $\left|\eta^{\text {jet } 3}\right| \leq 2.4$ and $E_{T}^{\text {jet }} \geq 6 \mathrm{GeV}$;
- $x_{\gamma}^{\text {obs }} \geq 0.75$.

To ensure that the tracks were well reconstructed and not associated with secondary charged particles generated via nuclear interactions within the detector material it was required that:

- the track transverse momentum was greater than 150 MeV ;
- the track pseudorapidity was between $\pm 1.7$;
- the track passed through at least 3 CTD super layers;
- the track was associated to the primary vertex.

The requirement that there be two and only two jets roughly balancing in $E_{T}^{\text {jet }}$ and in opposite hemispheres ensured that the events were LO-like, where the energy scale is well estimated using $M_{2 j} / 2$. The $x_{\gamma}^{\text {obs }}$ criterion was applied to minimise the influence of multiparton interactions (MPIs) [47-49], which generate additional final-state hadrons and can disrupt the correspondence between the MLLA predictions and the data. After all the above selection, the data sample contained 23,449 events.

## 7 Acceptance corrections

Effects due to the limited detector and trigger acceptance, efficiency and resolution were corrected for in the data using a sample of events generated with the Pythia Monte Carlo (MC) model [50, 51]. The direct and resolved photoproduction processes were generated separately and combined in the ratio that best fit the $x_{\gamma}^{\text {obs }}$ distribution in the data. The

Pythia model includes the $\mathrm{LO}(2 \rightarrow 2)$ matrix elements, approximates higher-order processes using initial-state and final-state parton showers and simulates hadronisation using the Lund string model [52]. The CTEQ5L [53] and GRV-G LO [54] parameterisations were used to describe the proton and photon PDFs, respectively. The main sample included MPIs, simulated using the "simple model" [50, 51] within Pythia, although the effects from MPIs were predicted to be negligible in the final sample. The detector simulation was based on Geant 3.21 [55] and included a complete simulation of the three-level trigger system.

The data were corrected bin-by-bin to the hadron-level using factors extracted from the MC equal to the ratio of the predicted hadron- to detector-level cross sections. Here, the hadron-level was defined to contain all particles with an average lifetime greater than 0.01 ns . The size of the bin-by-bin corrections were typically around 1.5 .

The normalisation of the $\xi$ distributions was set such that the integral of the distributions over the full $\xi$ range equalled $\left\langle N_{\text {jet }}^{\mathrm{ch}}\right\rangle$, where $\left\langle N_{\mathrm{jet}}^{\mathrm{ch}}\right\rangle$ denotes the average hadron-level charged-particle multiplicity within jets, with the appropriate cone and energy scale criteria applied. The values of $\left\langle N_{\text {jet }}^{\text {ch }}\right\rangle$ were extracted from the data by measuring the corresponding charged multiplicity distributions. These were corrected to the hadron-level using unfolding matrices derived from the Pythia MC sample. Full details of the procedure are described elsewhere [11, 56].

## 8 Systematic uncertainties

A detailed study [57] of the sources of systematic uncertainty associated with the measurement was performed. The dominant sources contributing to the systematic uncertainty on the $\xi$ distributions are listed below (the numbers in parentheses refer to the maximum uncertainty observed in any one bin):

- the $\pm 3 \%$ uncertainty in the CAL energy scale, propagated to the $\xi$ distributions by varying the CAL energies in the MC simulation accordingly ( $\pm 4 \%$ );
- the uncertainty simulating nuclear interactions in the detector material and the production of charged secondary particles. This was propagated to the $\xi$ distributions by varying the difference between the number of tracks gained and lost due to such effects in the MC by a factor of $2( \pm 4 \%)$;
- the uncertainty in the tracking efficiency, propagated to the $\xi$ distributions using the procedure described below ( $+5 \%$ ).

The MC slightly overestimated the number of tracks in the data, probably due to either the uncertainty in the hadronisation model or to inadequacies in the detector simulation. The unfolding procedure is only strongly sensitive to the detector-level simulation rather than the hadron-level MC model and it was assumed that this was the sole cause of the excess. This systematic uncertainty was evaluated by randomly failing detector-level tracks in the MC with track rejection rates evaluated in bins of $E_{\text {jet }}, \theta_{\text {trk }}$ and $1 / p_{\text {trk }}$, where $\theta_{\text {trk }}$ is the polar angle between the track and the jet axis. The largest rejection rate was $14 \%$. The
analysis was then repeated and the resulting difference in the $\xi$ distributions was included in the systematic uncertainty. All the systematic uncertainties were added in quadrature.

In the next section, several fits of the data are discussed. While nominally fitting the data and when evaluating the associated $\chi^{2}$ values, only the statistical uncertainties were considered. The systematic uncertainties on the data were propagated, however, to the fitted parameters using the "offset method". To apply the "offset method", the fit is repeated for each source of systematic uncertainty, shifting the nominal data by the uncertainty attributed to that one source. The differences between the values of the parameters extracted from the nominal and the shifted data are then summed in quadrature and included as the total systematic uncertainty on the parameter itself.

## 9 Results and discussion

The $\xi$ distributions were measured in five bins of $E_{\text {jet }}$ and in cones around the reconstructed jet axes with opening angles $\theta_{c}=\{0.23,0.28,0.34\}$. The characteristic energy scales of the five $E_{\text {jet }}$ bins, $E_{\text {jet }}=\{19,23,28,32,38\} \mathrm{GeV}$, were equated with the mean $E_{\text {jet }}$ value for all events contributing to that bin. They are shown in figure 1. Each of the distributions are observed to be similar in shape and are roughly Gaussian with more pronounced upper tails.

To assess the validity of the MLLA predictions using the measured $\xi$ distributions, two approaches were adopted. The first, discussed in section 9.1, was based solely on the position of the peak of the $\xi$ distributions, $\xi_{\text {peak }}$. The second was based on the full shape of the $\xi$ distributions and is discussed in section 9.2.

### 9.1 The $\xi_{\text {peak }}$ analysis

The values of $\xi_{\text {peak }}$ were extracted from the $\xi$ distributions using a three-parameter Gaussian fit. In accordance with previous analyses [11, 14], the distributions were fit in the range $\mu_{\xi} \pm 1$, where $\mu_{\xi}$ is the arithmetic mean of the $\xi$ distribution over the full $\xi$ range. The explicit ranges and $\chi^{2} /$ dof values of the fits are given in figure 1 . The $\chi^{2} /$ dof values range between 0.48 and 1.33 and hence indicate that the fits are reasonable.

Uncertainty in the $\xi_{\text {peak }}$ values due to the choice of fitting range was added in quadrature to the total systematic uncertainty. It was evaluated by changing the fit range to $\mu_{\xi} \pm 0.9$ and $\mu_{\xi} \pm 1.1$, leading maximally to a ${ }_{-1.31}^{+0.14} \%$ systematic effect. The largest and only other source contributing more than $1 \%$ to the systematic uncertainty was the CAL energy scale, leading to a ${ }_{-2.86}^{+0.58 \%} \%$ effect. The extracted values of $\xi_{\text {peak }}$ are given in table 1 and are observed to increase as the energy scale or $\theta_{c}$ increases.

The $\xi_{\text {peak }}$ values are shown in figure 2 as a function of $\mu \sin \left(\theta_{c}\right)$, where the characteristic energy scale here is $\mu=E_{\text {jet }}$. Also shown at their characteristic energy scales are data from the ZEUS ep DIS [11, 12] analysis and the OPAL [8], TASSO [10], NOMAD [13] and $\operatorname{CDF}[14]$ collaborations. There is an approximately linear relationship between $\xi_{\text {peak }}$ and $\ln \left(E_{\text {jet }} \sin \left(\theta_{c}\right)\right)$. This relationship was tested by fitting the $\xi_{\text {peak }}$ data, measured with $\theta_{c}=0.23$, with a straight line, parameterised as $\xi_{\text {peak }}=A\left(\ln \left(E_{\text {jet }} \sin \left(\theta_{c}\right)\right)\right)+B$. In the case where only the new ZEUS $\gamma p$ data were considered, the best fit values for the coefficients

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Figure 1. The $\xi$ distributions in the five $E_{\text {jet }}$ bins using the three $\theta_{c}$ values. The ZEUS data are shown by the solid squares. The inner error bars represent the statistical uncertainty. The outer error bars represent the statistical plus systematic uncertainties added in quadrature. Gaussian functions (solid line) have been fitted to the data within the regions indicated (dashed lines). The $\chi^{2} /$ dof of each fit is given on the plot.
were found to be $A=0.56 \pm 0.06$ (stat.) ${ }_{-0.03}^{+0.08}$ (syst.) and $B=1.16 \pm 0.09$ (stat.) ${ }_{-0.14}^{+0.06}$ (syst.). The $\chi^{2}$ /dof of the fit was 0.51 .

A test of the same linear relationship was made using the global data set in figure 2 . The best global fit values for the coefficients were found to be $A=0.682 \pm 0.007$ (stat. $\oplus$ syst.) and $B=1.009 \pm 0.019$ (stat. $\oplus$ syst.), with a $\chi^{2} /$ dof of 0.77 . Here, all systematic uncertainties were treated as uncorrelated. The globally-extracted parameters are consistent with those

| $E_{\text {jet }}(\mathrm{GeV})$ | $\theta_{c}$ | $\xi_{\text {peak }}$ | stat. | syst. |
| :---: | :---: | :---: | :---: | :---: |
| 19 | 0.23 | 1.99 | $\pm 0.01$ | ${ }_{-0.02}^{+0.02}$ |
|  | 0.28 | 2.10 | $\pm 0.01$ | ${ }_{-0.01}^{+0.01}$ |
|  | 0.34 | 2.20 | $\pm 0.01$ | ${ }_{-0.01}^{+0.01}$ |
| 23 | 0.23 | 2.11 | $\pm 0.02$ | ${ }_{-0.01}^{+0.02}$ |
|  | 0.28 | 2.21 | $\pm 0.02$ | ${ }_{-0.01}^{+0.02}$ |
|  | 0.34 | 2.32 | $\pm 0.02$ | ${ }_{-0.01}^{+0.02}$ |
| 28 | 0.23 | 2.22 | $\pm 0.04$ | ${ }_{-0.02}^{+0.03}$ |
|  | 0.28 | 2.34 | $\pm 0.03$ | ${ }_{-0.02}^{+0.02}$ |
|  | 0.34 | 2.44 | $\pm 0.04$ | ${ }_{-0.01}^{+0.04}$ |
| 32 | 0.23 | 2.25 | $\pm 0.07$ | ${ }_{-0.05}^{+0.09}$ |
|  | 0.28 | 2.36 | $\pm 0.06$ | ${ }_{-0.03}^{+0.10}$ |
|  | 0.34 | 2.56 | $\pm 0.06$ | ${ }_{-0.05}^{+0.07}$ |
| 38 | 0.23 | 2.40 | $\pm 0.05$ | ${ }_{-0.08}^{+0.04}$ |
|  | 0.28 | 2.50 | $\pm 0.08$ | ${ }_{-0.18}^{+0.07}$ |
|  | 0.34 | 2.59 | $\pm 0.07$ | +0.18 +0.08 -0.15 |

Table 1. $\xi_{\text {peak }}$ values in the five $E_{\text {jet }}$ bins using the three $\theta_{c}$ values. The statistical and systematic uncertainties are also given.

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Figure 2. $\xi_{\text {peak }}$ as a function of $\mu \sin \left(\theta_{c}\right)$, where $\mu$ denotes the characteristic energy scale for each specific process. The ZEUS $\gamma p$ data (solid circles) are shown along with $e p$ data from the ZEUS collaboration (diamonds) and results reported by the OPAL (crosses), TASSO (triangles), NOMAD (stars) and CDF (open circles) collaborations. The inner error bars on the ZEUS points represent the statistical uncertainty. The outer error bars represent the statistical plus systematic uncertainties added in quadrature for all data sets. The data have been fitted with a straight line.

|  |  | $\xi_{\text {peak }}$ analysis |  |  | $\xi$ shape analysis |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $E_{\text {jet }}(\mathrm{GeV})$ | $\theta_{c}$ | $\Lambda_{\text {eff }}(\mathrm{MeV})$ | stat. | syst. | $\Lambda_{\text {eff }}(\mathrm{MeV})$ | stat. | syst. |
| 19 | 0.23 | 272 | $\pm 5$ | ${ }_{-8}^{+6}$ | 304 | $\pm 4$ | ${ }_{-32}^{+7}$ |
|  | 0.28 | 280 | $\pm 4$ | ${ }_{-5}^{+5}$ | 298 | $\pm 4$ | +21 +25 |
|  | 0.34 | 289 | $\pm 4$ | ${ }_{-5}^{+6}$ | 303 | $\pm 3$ | +15 +15 +31 |
| 23 | 0.23 | 280 | $\pm 7$ | ${ }_{-7}^{+6}$ | 307 | $\pm 6$ | ${ }_{+}^{+10}$ |
|  | 0.28 | 291 | $\pm 9$ | ${ }_{-11}^{+3}$ | 305 | $\pm 6$ | +32 +32 -23 |
|  | 0.34 | 297 | $\pm 8$ | +31 +9 +9 | 301 | $\pm 5$ | +32 +26 +29 |
| 28 | 0.23 | 279 | $\pm 16$ | ${ }_{-11}^{+8}$ | 285 | $\pm 12$ | ${ }_{-19}^{+8}$ |
|  | 0.28 | 282 | $\pm 14$ | ${ }_{-9}^{+8}$ | 294 | $\pm 10$ | ${ }_{-}^{+7}$ |
|  | 0.34 | 292 | $\pm 17$ | ${ }_{-17}^{+5}$ | 287 | $\pm 9$ | +29 +23 |
| 32 | 0.23 | 310 | $\pm 33$ | ${ }_{+}^{+22}$ | 298 | $\pm 15$ | +25 +40 -29 |
|  | 0.28 | 321 | $\pm 29$ | +41 +14 -49 | 302 | $\pm 13$ | -40 <br> +26 <br> -21 |
|  | 0.34 | 283 | $\pm 24$ | -49 +21 +28 | 286 | $\pm 14$ | -41 +28 +27 |
| 38 | 0.23 | 290 | $\pm 23$ | ${ }_{-16}^{+38}$ | 311 | $\pm 15$ | ${ }_{+}^{+13}$ |
|  | 0.28 | 301 | $\pm 37$ | +-168 +48 -38 | 287 | $\pm 21$ | +52 +42 +32 |
|  | 0.34 | 319 | $\pm 36$ | +33 +31 +38 | 297 | $\pm 17$ | -32 <br> +21 <br> + |

Table 2. $\Lambda_{\mathrm{eff}}$ extracted at the five $E_{\text {jet }}$ points using the three $\theta_{c}$ values obtained from both the $\xi_{\text {peak }}$ and $\xi$ shape analyses. The statistical and systematic uncertainties are also given.
extracted from the ZEUS data alone. The ZEUS $\gamma p$ points are systematically below the global-fit line, however the differences are within the total experimental uncertainty.

The MLLA in fact predicts a small square-root correction to the perfect linear dependence, as seen in eq. (2.3). Assuming $\Lambda_{\text {eff }}$ is constant within the range of energies probed, eq. (2.3) can be directly fit to the $\xi_{\text {peak }}$ data, treating $\Lambda_{\text {eff }}$ as a free parameter. In the case where only the ZEUS $\gamma p$ data with $\theta_{c}=0.23$ were considered, the best fit value was found to be $\Lambda_{\text {eff }}=275 \pm 4$ (stat. $)_{-8}^{+4}$ (syst.) MeV . The $\chi^{2} /$ dof of the fit was 0.70 , indicating a good fit. When the global data set was considered, the best fit value was found to be $\Lambda_{\text {eff }}=246 \pm 3$ (stat. $\oplus$ syst.) MeV. In the global fit, all uncertainties were treated as uncorrelated. The $\chi^{2}$ /dof of the fit, with this simplistic error treatment, was 2.2 , indicating some discrepancy. The globally extracted value of $\Lambda_{\text {eff }}$ is not consistent with that extracted from the ZEUS data alone.

The energy dependence of $\Lambda_{\text {eff }}$ was studied by using eq. (2.3) to map each $\xi_{\text {peak }}$ value to a corresponding value of $\Lambda_{\text {eff }}$. The results, given in table 2 and shown in figure 3 as a function of $E_{\text {jet }}$, show no evidence that $\Lambda_{\text {eff }}$ is dependent on the energy scale. A weak dependence was observed in the CDF data [14], which span a wider range of energy scales. However, the data do suggest that the value of $\Lambda_{\mathrm{eff}}$ is weakly dependent on $\theta_{c}$. Specifically, figure 3 shows that the values of $\Lambda_{\text {eff }}$ extracted from the wider cone data tend to be systematically larger. This behaviour was also observed by the CDF collaboration [14]. Both the $\theta_{c}$ and $E_{\text {jet }}$ dependence seen by CDF would contribute to the discrepancy observed when fitting eq. (2.3) to the global data set.

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Figure 3. $\Lambda_{\text {eff }}$ extracted at the five $E_{\text {jet }}$ points using the three $\theta_{c}$ values. The ZEUS data are shown by the solid points. The inner error bars represent the statistical uncertainty. The outer error bars represent the statistical plus systematic uncertainties added in quadrature. The points have been shifted horizontally for clarity.

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Figure 4. $\Lambda_{\text {eff }}$ as a function of $\mu$, where $\mu$ denotes the characteristic energy scale for each specific process. The ZEUS $\gamma p$ data are shown by the solid circles. Also shown are $e p$ data from the ZEUS collaboration and results reported by the OPAL, L3 and CDF collaborations. The inner error bars on the ZEUS $\gamma p$ points represent the statistical uncertainty. The outer error bars represent the statistical plus systematic uncertainties added in quadrature for all data sets.

In figure 4 , the values of $\Lambda_{\text {eff }}$ extracted using the $\xi_{\text {peak }}$ data are shown as a function of the energy scale and compared to the previous results from ZEUS [11] using ep DIS collisions, and the OPAL [8], L3 [9] and CDF [14] collaborations. The values are all largely consistent in the energy scale region shown, supporting the prediction that $\Lambda_{\text {eff }}$ is a universal parameter.

### 9.2 The $\xi$-shape analysis

The $\xi$ distributions were also fitted using the predicted limiting spectrum, according to eq. (2.4). The quantities $K$ and $\Lambda_{\text {eff }}$ were treated as free parameters during the fit. The fitted MLLA functions are shown in figure 5 . The fits were restricted to the ranges indicated by the vertical lines and the $\chi^{2}$ /dof values of the fits are also given and lie between 0.34 and 2.72. Typically, in each $E_{\text {jet }}$ bin, the $\chi^{2} /$ dof increases as $\theta_{c}$ does. The $\chi^{2} /$ dof values indicate that, while the theory does describe many of the features of the data in the fitting ranges, there are differences. Specifically, the rising edges of the $\xi$ peaks are well described. However, the upper tails of the distributions are not adequately reproduced. The same was observed in $e^{+} e^{-}[8,9]$ and $e p$ DIS [11] data and to a lesser extent in high- $E_{\text {jet }} p \bar{p}$ data [14]. This is likely due to the specific MLLA regularisation scheme used here and in the other aforementioned analyses.

As discussed in section 2, the MLLA regularisation scheme used here causes the partons to be cut-off at $p_{T}^{\mathrm{rel}, \mathrm{pl}}=\Lambda_{\text {eff }}$, whereas the hadrons in the data are not. This leads to an intrinsic discrepancy between data and theory. The discrepancy is present for all $\xi>0$, however the magnitude of the effect is small at low $\xi$ and increases until, for all $\xi>$ $\ln \left(E_{\text {jet }} \sin \left(\theta_{c}\right) / \Lambda_{\text {eff }}\right)$, there are only hadrons and no partons.

A consequence of this discrepancy is that, in order to fit the data using eq. (2.4), a relatively arbitrary upper fitting bound, $\xi_{+}$, had to be chosen for each $\xi$ distribution. The criteria used to set $\xi_{+}$were that the resulting fits were reasonably stable and that $\xi_{\text {peak }} \ll \xi_{+}<\ln \left(E_{\text {jet }} / 250 \mathrm{MeV}\right)$ was satisfied, where 250 MeV roughly corresponds to the values of $\Lambda_{\text {eff }}$ extracted from the $\xi_{\text {peak }}$ data. The finite experimental $\xi$ binning was also a consideration. It was chosen to use $\xi_{+}=w \xi_{\text {peak }}+(1-w) \ln \left(E_{\text {jet }} / 250 \mathrm{MeV}\right)$, with $w=0.25$ for the nominal fits. The sensitivity of $K$ and $\Lambda_{\text {eff }}$ to the choice of the fitting range was treated as a systematic uncertainty and was evaluated by varying $w$ by $\pm 0.1$. This source of uncertainty strongly dominates the overall uncertainty on $\Lambda_{\text {eff }}$, leading to $\mathrm{a}_{-10.6}^{+1.8} \%$ effect, although $K$ was found to be largely insensitive to it. The same lower fitting bound, $\xi_{-}=\ln (2)$, was used is all cases and both $K$ and $\Lambda_{\text {eff }}$ were observed to be insensitive to a variation of $\xi_{-}$by $\pm 15 \%$.

The values of $\Lambda_{\text {eff }}$ extracted from the MLLA fits are given in table 2. The results are in reasonable agreement with those extracted from the $\xi_{\text {peak }}$ data, although the values extracted using the MLLA fit have larger uncertainties. The value of $\Lambda_{\text {eff }}$ from the MLLA method with $\theta_{c}=0.23$ and averaged over $E_{\text {jet }}$, weighting each data point based only on its statistical precision, is $\Lambda_{\text {eff }}=304 \pm 6$ (stat.) ${ }_{-32}^{+8}$ (syst.) MeV.

Values of $\kappa_{\text {ch }}$ were extracted from the fitted $K$ values using eq. (2.4) and the values of $\epsilon_{\mathrm{g}}$ predicted for each $E_{\mathrm{jet}}$ bin by the PythiA model. The $\epsilon_{\mathrm{g}}$ values were roughly constant in $E_{\mathrm{jet}}$, at $\epsilon_{\mathrm{g}} \approx 0.2$. The $\kappa_{\mathrm{ch}}$ values are given in table 3 and are shown in figure 6 . The

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Figure 5. The $\xi$ distributions in the five $E_{\text {jet }}$ bins using the three $\theta_{c}$ values. The ZEUS data are shown by the solid squares. The inner error bars represent the statistical uncertainty. The outer error bars represent the statistical plus systematic uncertainties added in quadrature. The limited momentum spectrum predicted by the MLLA (solid line) has been fitted to the data within the regions indicated (dashed lines). The $\chi^{2} /$ dof of each fit is given on the plot.
total uncertainty is dominated by the theoretical uncertainty associated with the next-toMLLA correction factors. The $\kappa_{\mathrm{ch}}$ data suggest a weak dependence on $\theta_{c}$. Specifically, as $\theta_{c}$ increases, so too does the central value of $\kappa_{\mathrm{ch}}$. This is significant when the high degree of statistical correlation between the three $\theta_{c}$ samples and the bin-to-bin correlation in the systematic and theoretical uncertainties are taken into consideration. The same is true for the $\kappa_{\text {ch }}$ values reported by the CDF collaboration [14], which were obtained using a

| $E_{\text {jet }}(\mathrm{GeV})$ | $\theta_{c}$ | $\kappa^{\text {ch }}$ | stat. | syst. | theo. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 19 | 0.23 | 0.54 | $\pm 0.01$ | ${ }_{-0.02}^{+0.03}$ | ${ }_{-0.09}^{+0.11}$ |
|  | 0.28 | 0.59 | $\pm 0.01$ | ${ }_{-0.01}^{+0.03}$ | ${ }_{-0.10}^{+0.12}$ |
|  | 0.34 | 0.63 | $\pm 0.01$ | ${ }_{-0.02}^{+0.03}$ | ${ }_{-0.10}^{+0.12}$ |
| 23 | 0.23 | 0.56 | $\pm 0.01$ | ${ }_{-0.02}^{+0.03}$ | ${ }_{-0.09}^{+0.11}$ |
|  | 0.28 | 0.60 | $\pm 0.01$ | ${ }_{-0}^{+0.04}$ | ${ }_{-0.10}^{+0.12}$ |
|  | 0.34 | 0.63 | $\pm 0.01$ | +0.04 | -0.10 +0.13 |
|  |  | 0.63 | $\pm 0.01$ | -0.02 | -0.10 |
| 28 | 0.23 | 0.55 | $\pm 0.01$ | ${ }_{-0.01}^{+0.04}$ | ${ }_{-0.09}^{+0.11}$ |
|  | 0.28 | 0.59 | $\pm 0.01$ | ${ }_{-0.04}^{+0.04}$ | ${ }_{-0.09}^{+0.11}$ |
|  | 0.34 | 0.61 | $\pm 0.01$ | ${ }_{-}^{+0.04}$ | ${ }_{-}^{+0.12}$ |
| 32 | 0.23 | 0.56 | $\pm 0.02$ | +0.04 | +0.11 |
|  | 0.23 | 0.56 | $\pm 0.02$ | -0.04 | -0.09 |
|  | 0.28 | 0.59 | $\pm 0.02$ | ${ }_{-0.04}^{+0.04}$ | ${ }_{-0.09}^{+0.11}$ |
|  | 0.34 | 0.61 | $\pm 0.02$ | ${ }_{-0.03}^{+0.04}$ | ${ }_{-0.10}^{+0.12}$ |
| 38 | 0.23 | 0.56 | $\pm 0.03$ | ${ }_{-}^{+0.05}$ | ${ }_{-}^{+0.11}$ |
|  | 0.28 | 0.58 | +0.03 | -0.06 +0.04 | -0.09 |
|  | 0.28 | 0.58 | $\pm 0.03$ | -0.04 | -0.09 |
|  | 0.34 | 0.61 | $\pm 0.03$ | ${ }_{-0.05}^{+0.03}$ | ${ }_{-0.10}^{+0.12}$ |

Table 3. $\kappa^{\mathrm{ch}}$ values extracted at the five $E_{\text {jet }}$ points using the three $\theta_{c}$ values. The statistical, systematic and theoretical uncertainties are also given.

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Figure 6. $\kappa^{\mathrm{ch}}$ extracted at the five $E_{\text {jet }}$ points using the three $\theta_{c}$ values. The ZEUS data are shown by the solid points. The inner error bars represent the statistical uncertainty. The outer error bars represent the statistical, systematic and theoretical uncertainties added in quadrature. The points have been shifted horizontally for clarity.
different extraction method. The ZEUS data in figure 6 do not provide any evidence that $\kappa_{\mathrm{ch}}$ is dependent on $E_{\text {jet }}$ in the range probed.

The value of $\kappa_{\mathrm{ch}}$, measured with $\theta_{c}=0.23$ and averaged over $E_{\text {jet }}$, weighting the data points based on their statistical precision, was $\kappa_{\mathrm{ch}}=0.55 \pm$ 0.01 (stat. $)_{-0.02}^{+0.03}(\text { syst. })_{-0.09}^{+0.11}$ (theo.). The $\kappa_{\text {ch }}$ value extracted here is in good agreement with that reported by the CDF collaboration, $\kappa_{\mathrm{ch}}=0.56 \pm 0.05$ (stat.) $\pm 0.09$ (syst.). To compare to the values extracted using $e^{+} e^{-}$and $e p$ DIS data and assuming no contamination from gluon jets, the values have to be scaled by $r C_{F} / F_{\mathrm{nMLLA}} C_{A} \approx 0.55$. This leads to values of $\kappa_{\mathrm{ch}} \approx 0.7$. These other results were found with $\theta_{c}$ effectively set to $\pi / 2$ however. In addition, the differing proportion of heavy- to light-flavour quarks in each environment, not accounted for in the MLLA or LPHD, makes the comparisons inexact.

## 10 Summary

The multiplicity distributions of charged particles within cones centred on jets have been measured as a function of $\xi=\ln \left(1 / x_{p}\right)$, where $x_{p}$ is the fraction of the jet's momentum carried by the charged particle. These $\xi$ distributions have been measured in five bins of $E_{\text {jet }}$ and with three different cone opening angles, $\theta_{c}$, for $\gamma p$ events containing two and only two jets, using $359 \mathrm{pb}^{-1}$ of ep data.

The peak positions of the $\xi$ distributions, $\xi_{\text {peak }}$, were extracted and observed to increase roughly linearly with $\ln \left(E_{\text {jet }} \sin \left(\theta_{c}\right)\right)$. A single value of intrinsic MLLA scale, $\Lambda_{\text {eff }}$, was extracted by fitting the $\xi_{\text {peak }}$ data according to the predicted relationship between $\xi_{\text {peak }}$ and $\ln \left(E_{\text {jet }} \sin \left(\theta_{c}\right) / \Lambda_{\text {eff }}\right)$. The best fit value was found to be $\Lambda_{\text {eff }}=275 \pm 4$ (stat. $)_{-8}^{+4}$ (syst.) MeV .

The $E_{\text {jet }}$ and $\theta_{c}$ dependences of $\Lambda_{\text {eff }}$ were studied by calculating a value of $\Lambda_{\text {eff }}$ from each $\xi_{\text {peak }}$ data point. The value of $\Lambda_{\text {eff }}$ weakly depends on $\theta_{c}$ but no $E_{\text {jet }}$ dependence was observed. The $\Lambda_{\text {eff }}$ data are consistent with previously published data sets using different initial states, supporting the prediction that $\Lambda_{\text {eff }}$ is universal.

The $\xi$ distributions were also fitted using the limited momentum spectra predicted by the MLLA and assuming LPHD, in the regions where they are applicable. The theory largely described the data in these regions. The fitted MLLA functions were used to extract the value of $\Lambda_{\mathrm{eff}}$ as a function of $E_{\mathrm{jet}}$ and $\theta_{c}$. The value extracted using this method with $\theta_{c}=0.23$ and averaged over $E_{\text {jet }}$, was $\Lambda_{\text {eff }}=304 \pm 6(\text { stat. })_{-32}^{+8}$ (syst.) MeV .

The value of the LPHD parameter $\kappa_{\mathrm{ch}}$ was extracted as a function of $E_{\mathrm{jet}}$ and $\theta_{c}$ from the fitted limited momentum spectra. Corrections based on next-to-MLLA theory were included. The value extracted with $\theta_{c}=0.23$ and averaged over $E_{\mathrm{jet}}$, was $\kappa_{\mathrm{ch}}=$ $0.55 \pm 0.01$ (stat.) ${ }_{-0.02}^{+0.03}$ (syst.) ${ }_{-0.09}^{+0.11}$ (theo.). The value of $\kappa_{\text {ch }}$ has a weak dependence on $\theta_{c}$ and is consistent with the results published by the CDF collaboration. The data support the assumption that $\kappa_{\text {ch }}$ is universal.

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[^0]:    ${ }^{1}$ Attempts using Monte Carlo models to account for heavy-flavour effects have been made elsewhere [27].
    ${ }^{2}$ The word "electron" is used as a generic term for electrons and positrons.
    ${ }^{3}$ The ZEUS coordinate system is a right-handed Cartesian system, with the $Z$ axis pointing in the proton beam direction, referred to as the "forward direction", and the $X$ axis pointing towards the centre of HERA. The coordinate origin is at the nominal interaction point.

